

# Graph Theory and Qubit Information Systems of Extremal Black Branes

Adil Belhaj<sup>1</sup>, Moulay Brahim Sedra<sup>2,3</sup>, Antonio Segui<sup>4</sup>

<sup>1</sup>Département de Physique, Faculté Polydisciplinaire, Université Sultan Moulay Slimane  
Béni Mellal, Morocco

<sup>2</sup> LHESIR, Département de Physique, Faculté des Sciences, Université Ibn Tofail  
Kénitra, Morocco

<sup>3</sup> Université Mohammed Premier, Ecole Nationale des Sciences Appliquées  
BP : 3, Ajdir, 32003, Al Hoceima, Morocco

<sup>4</sup> Departamento de Física Teórica, Universidad de Zaragoza, E-50009-Zaragoza, Spain.

## Abstract

Using graph theory based on Adinkras, we consider once again the study of extremal black branes in the framework of quantum information. More precisely, we propose a one to one correspondence between qubit systems, Adinkras and certain extremal black branes obtained from type IIA superstring compactified on  $T^n$ . We accordingly interpret the real Hodge diagram of  $T^n$  as the geometry of a class of Adinkras formed by  $2^n$  bosonic nodes representing  $n$  qubits. In this graphic representation, each node encodes information on the qubit quantum states and the charges of the extremal black branes built on  $T^n$ . The correspondence is generalized to  $n$  superqubits associated with odd and even geometries on the real supermanifold  $T^{n|n}$ . Using a combinatorial computation, general expressions describing the number of the bosonic and the fermionic states are obtained.

**Keywords:** String theory, extremal black holes and branes, graph theory, Andinkras, qubit information systems and supermanifolds.

# 1 Introduction

Recently, extremal black branes in arbitrary dimensions have been investigated using different methods in the framework of string theory and related topics including M-theory compactified on manifolds having special holonomy groups [1, 2, 3, 4]. These black objects have been studied using the so called attractor mechanism [5, 6, 7]. In this way, the scalar fields can be fixed in terms of the black brane charges. This analysis can be done in terms of an effective potential depending on the charges and the stringy moduli obtained from the compactification of higher dimensional theories. Extremising the potential with respect to such moduli, the minimum generates the scalar fixed values. Moreover, the corresponding entropy functions have been computed using the U-duality symmetry acting on the invariant black brane charges of the compactified theories. In this regard, the Calabi-Yau compactifications have been explored to produce several interesting results dealing with the black objects in higher dimensional theories including string theory [8, 9, 10, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18].

More recently, a connection with quantum information has been proposed using the qubit formalism[19, 20, 21, 22, 23, 24]. It is recalled that the qubit is the building piece of quantum information theory. In fact, a possible interplay between the STU black holes having eight charges and three qubits have been given in [25, 26]. The analysis has been extended to include superqubits using supersymmetric approach based on the theory of Lie superalgebras. In particular, the  $osp(2|1)$  Lie superalgebra has been explored to deal with the physics of the superqubits[27, 28, 29].

In this paper, we reconsider the study of the extremal black branes in the framework of quantum information using graph theory based on the so called Adinkras [32, 33]. More precisely, we establish a one to one correspondence between qubit systems, Adinkras and extremal black branes embedded in maximally supersymmetric supergravity obtained from a low energy limit of type IIA superstring compactified on  $T^n$ . We accordingly interpret the real Hodge diagram of  $T^n$  as the geometry of a class of Adinkras involving  $2^n$  bosonic nodes representing  $n$  qubits. In this graphic representation, each node encodes information on the qubit quantum states and the charges of the extremal black branes constructed from the compactification of type IIA superstring on  $T^n$ . The correspondence is generalized to  $n$  superqubits associated with odd and even geometries on the real supermanifold  $T^{n|n}$ . Using combinatorial computation, general expressions describing the number of bosonic and fermionic states are obtained. Then, illustrated models are given. It is worth noting that another nice connection between cohomology of extra dimensions and qubit systems have been also given in [24, 31]

The organization of the paper is as follows. In section 2, we reconsider the study of the extremal black branes in type IIA superstring compactified on  $T^n$ . Section 3 concerns an elaboration of a correspondence between qubit systems, Andinkras and the real Hodge diagrams of  $T^n$ . In section 4, we give a graphic representation of the extremal black branes using qubit systems based on Adinkras. The generalization to superqubit systems is given in section 5. The analysis is done in terms of the real supermanifold  $T^n|n$ . The last section is devoted to the conclusion and open questions.

## 2 Extremal black branes in string theory

In this section, we reconsider the discussion of the extremal black branes in string theory compactified on  $n$ -dimensional compact manifolds  $X^n$ . It has been shown that the black  $p$ -objects can be produced by a system of  $(p+k)$ -branes wrapping the  $k$ -cycles of  $X^n$ . In such a compactification, it has been shown that the near horizon geometries of the extremal black  $p$ -brane are defined by the product of Ads spaces and spheres

$$AdS_{p+2} \times S^{8-n-p}. \quad (2.1)$$

It is obvious to see that the integers  $n$  and  $p$  should satisfy

$$1 \leq n, \quad 2 \leq 8 - n - p. \quad (2.2)$$

The generalization of such geometries to investigate intersecting attractors has been also given in [18].

In higher dimensional theories, one may classify the the black  $p$ -brane solutions using the extended electric/magnetic duality connecting a  $p$ -dimensional electrical black brane to a  $q$ -dimensional magnetic one via the following relation

$$p + q = 6 - n. \quad (2.3)$$

The solution of this equation provides three different black objects organized in terms of the values of the charge couple  $(p, q)$ . Indeed, they are given by

- $(p, q) = (0, 6 - n)$ , associated with the electrical charged black holes,
- $(p, q) = (3 - \frac{n}{2}, 3 - \frac{n}{2})$  ( $n$  even), corresponding to the dyonic black branes, having both the electric and magnetic charges,
- $(p, q) \neq (0, 6 - n)$ , where  $(p, q) \neq (3 - \frac{n}{2}, 3 - \frac{n}{2})$ , describing the magnetic black branes.

It is noted that  $p = 0$  describes the electric charged black holes in  $10 - n$  dimensions. Their dual magnetic are black  $6 - n$ -branes. These black objects carry charges corresponding to

the gauge invariant field strengths ( $F = dA$ ) of the corresponding maximally supersymmetric supergravity theory. It is worth recalling that even dimensional space-times produce dyonic black branes. The charges of these objects can fix the value of the dilaton using the attractor mechanism as reported in [17].

Before examining a particular supergravity theory solution, we should recall that the black object charges depend on the choice of the internal space. It turns out that, each compactification involves a Hodge diagram carrying not only geometric information but also physical data on the black brane solutions.

In the following sections, we will be concerned with the compactification of type IIA superstring theory on the  $n$ -dimensional tori  $T^n$ . This compactification may produce the black  $p$ -brane configurations in  $10 - n$  dimensional maximally supersymmetric supergravity coupled to abelian gauge symmetries associated with the **NS-NS** and **R-R** bosonic fields of various ranks. In this way, the black objects can be constructed using the following brane configurations

$$\text{D0-branes, F-strings, D2-branes, D4-branes, NS-5branes, D6-branes.}$$

As in the case of the Calabi-Yau manifold, the toroidal compactification involves a real Hodge diagram playing a primordial role in the elaboration of the type IIA superstring charge spectrum in  $10 - n$  dimensions. Roughly speaking,  $T^n$  is a flat compact space which can be defined using different ways. One of them is to use the trivial fibration of  $n$  circles modeled by the following orbital relations

$$x_i \equiv x_i + 1, \quad i = 1, \dots, n. \quad (2.4)$$

To build the real Hodge diagrams, it is convenient to introduce a binary number notation  $h^{e_1, \dots, e_n}$  playing a similar role as the Hodge numbers appearing in the cohomology of complex Calabi-Yau geometries. More precisely, this number is associated with the following differential form

$$h^{e_1, \dots, e_n} \longrightarrow \bigwedge_{\ell=1}^n (\overline{e}_\ell + e_\ell dx_\ell), \quad (2.5)$$

where  $e_\ell$  is a binary number taking either 0 or 1, and where  $\overline{e}_\ell$  is its conjugate. It is obvious to see that  $\prod_{\ell=1}^n (\overline{e}_\ell + e_\ell dx_\ell)$  is a real differential form of degree  $k$  which can be written as follows

$$k = \sum_{\ell=1}^n e_\ell. \quad (2.6)$$

By using the Poincaré duality, this  $k$ -form is dual to a  $k$ -cycle embedded in  $T^n$  on which type IIA branes could wrap to produce black objects in  $10 - n$  dimensions. By inverting the order of the Hodge diagram corresponding to the Calabi-Yau manifolds, we can give a graphic representation of the cohomology space of  $T^n$  in terms of the real Hodge diagrams. For

simplicity reason, we illustrate the  $n = 2$  and  $n = 3$  cases

$n = 2$	$h^{1,1}$	1
	$h^{1,0} \quad h^{0,1}$	1      1
	$h^{0,0}$	1
$n = 3$	$h^{1,1,1}$	1
	$h^{1,1,0} \quad h^{1,0,1} \quad h^{0,1,1}$	1   1   1
	$h^{1,0,0} \quad h^{0,1,0} \quad h^{0,0,1}$	1   1   1
	$h^{0,0,0}$	1

It is noted that the general real Hodge diagram associated with  $T^n$  can be constructed by using the above mentioned notation and terminology. It encodes all possible non trivial cycles of  $T^n$  describing its geometric data including the size and the shape parameters.

A close inspection shows that there is a striking resemblance between the real Hodge diagram of  $T^n$  and graph theory of a particular class of Adinkras. This leads to a nice correspondence between the nodes of such a class of Adinkras and the cycles involved in the determination of the black brane charges in type IIA superstring on  $T^n$ . On the other hand, these Adinkras have been explored to give a graphic representation of qubit systems. Naturally, there should be a relation between all these issues. This question will be addressed in the following sections.

### 3 Qubit systems, Andinkras and real Hodge diagrams

In this section, we would like to elaborate a link between qubits, Adinkras and the real Hodge diagrams of  $T^n$ . This correspondence will be explored to engineer graphically the extremal black brane geometries using qubit systems. It is recalled that the qubit is a primordial piece in quantum information theory which have been extensively investigated using different physical and mathematical approaches [34, 35, 36]. It is a two configuration system which can be associated, for instance, with the electron in the hydrogen atom. The general state of a single qubit is usually given by the Dirac notation as follows

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle \quad (3.1)$$

where  $c_i$  are complex coefficients satisfying the normalization condition

$$|c_0|^2 + |c_1|^2 = 1. \quad (3.2)$$

This equation can be interpreted geometrically in terms of the so called Bloch sphere. The two qubits are four configuration systems. In this case, the general state takes the following form

$$|\psi\rangle = c_{00}|00\rangle + c_{10}|10\rangle + c_{01}|01\rangle + c_{11}|11\rangle \quad (3.3)$$

where  $c_{ij}$  are complex numbers satisfying the normalization condition

$$|c_{00}|^2 + |c_{10}|^2 + |c_{01}|^2 + |c_{11}|^2 = 1, \quad (3.4)$$

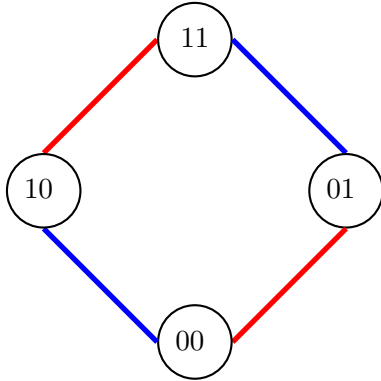
describing a three dimensional complex projective space  $CP^3$  generalizing the Bloch sphere. This analysis can be extended to  $n$  qubits associated with  $2^n$  configuration states using the above binary notation.

It is observed that the qubit systems can be represented by diagrams having a strong resemblance with a particular class of Adinkras. For later, we refer to them as bosonic graphs. The Adinkras have been introduced in the study of supersymmetric representation theory [32, 33, 37, 38, 39, 40, 41]. As the usual diagram representation, Adinkras are formed by nodes and lines. It has been shown that there are various classes which have been explored in the classification of supersymmetric theories. These diagrams contain bosonic and fermionic nodes like Dynkin diagrams of Lie superalgebras. A particular graph is called regular formed by  $2^n$  nodes connected with  $n$  colored lines. In fact, this graph has been explored to represent  $n$ -qubits. More specifically, to each node, one associates a state of the qubit. To be precise, the correspondence can be formulated as follows

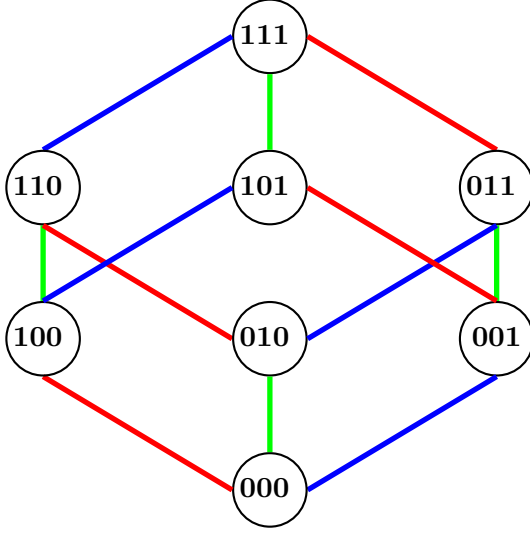
$$\text{node} \longrightarrow \text{state} \quad (3.5)$$

$$\text{number of colors} \longrightarrow \text{number of qubits} \quad (3.6)$$

To illustrate, we present two bosonic graphs associated with  $n = 2$  and  $n = 3$ . The first example concerns  $n = 2$  qubits and it is given by



The second example describes  $n = 3$  qubits and it is represented by



An inspection shows that the real Hodge diagrams of  $T^n$  can be interpreted as a regular Adinkra. To each node, we associate then a single cycle in  $T^n$ . As we will see shortly in an example, this link is as follows

$$h^{e_1, \dots, e_n} \longrightarrow \text{node} = (e_1, \dots, e_n). \quad (3.7)$$

The number of the nodes in the  $k$ -level is exactly the combinatorial number

$$\text{nbr}(k\text{-level nodes}) = C_n^k \quad (3.8)$$

indicating also the number of the  $k$ -cycle in  $T^n$ . The total number of the cycles is identified with the total number of the nodes. This number is given by

$$\text{nbr}(\text{cycles}) = \sum_{k=0}^n C_n^k = 2^n. \quad (3.9)$$

On the basis of the above link, the cycles in  $T^n$  should be associated with the states defining the  $n$  qubit systems. In this way, a quantum state is interpreted as the Poincaré dual of the real homology cycle on which branes can wrap to produce a black object in  $10 - n$  dimensional type IIA superstring. We expect that the Adinkras should encode information on the black brane physics in higher dimensional supersymmetric supergravity theories. This may offer a new take on the graphical representation of such a physics using techniques of graph theory. The following parts concern graphic representations of the extremal black branes using qubit and Adinkra notions.

## 4 Extremal black branes and Andinkra representation of qubit systems

In this section, we examine the correspondence between the extremal black  $p$ -branes in maximally supersymmetric supergravity and qubit systems using a graph theory based on bosonic Adinkras. To understand how such a surprising connection could be true, we first consider the case of the elliptic curve. Then, we give a general picture which may appear in lower dimensional theories.

### 4.1 The elliptic curve compactification

As mentioned, we start by discussing the eight dimensional extremal black branes. In particular, we will give two explicit models corresponding to  $p = 0$  and  $p = 4$ . These models can be obtained from the compactification of type IIA superstring on  $T^2$ , considered as a trivial fibration of two circles:  $S^1 \times S^1$ . The compactification of the massless bosonic ten dimensional type IIA superstring fields

$$NS - NS : G_{MN}, B_{MN}, \phi \quad R - R : A_M, C_{MNK} \quad M, N, K = 0, \dots, 9 \quad (4.1)$$

gives an eight dimensional spectrum encoding also information on the black brane charges. This spectrum, which can be alternatively obtained from the compactification of M-theory on  $T^3$  with the  $SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$  U-duality group [42], contains a graviton, three 2-form gauge fields, six vector gauge fields, one self dual 3-form gauge field and seven scalar fields. The associated scalar manifold reads as

$$\frac{SL(3, \mathbb{R})}{SO(3)} \times \frac{SL(2, \mathbb{R})}{SO(2)}. \quad (4.2)$$

Motivated by the study of the attractors on the K3 surface in type IIA superstring, another factorization of such a moduli space has been given in [16]. In type IIA six dimensions obtained from the compactification of the K3 surface, the moduli space contains two factors associated with two possible brane charges given by black strings and holes. According to [16], the separation of the extremal black brane charges in eight dimensions provides a possible factorization of the above scalar manifold given by

$$\frac{SO(2, 2)}{SO(2) \times SO(2)} \times \frac{SO(2, 1)}{SO(2)} \times SO(1, 1). \quad (4.3)$$

associated with three possible black brane charges. In this eight dimensional  $N = 2$  supergravity, the extremal near-horizon geometries of the black  $p$ -branes take the following form [18]

$$AdS_{p+2} \times S^{6-p}. \quad (4.4)$$

Indeed, they are classified into three categories:





$T^2$	Adinkra	qubit system	Black hole system	Black 4-brane system
1	(00)	$ 00\rangle$	D0-brane	D4-brane
$dx_1$	(01)	$ 01\rangle$	F-string	NS5-brane
$dx_2$	(10)	$ 10\rangle$	F-string	NS5-brane
$dx_1 dx_2$	(11)	$ 11\rangle$	D2-brane	D6-brane

Table 1: This table gives the correspondence between the eight dimensional black branes, Adinkra and qubit systems.

## 4.2 Lower dimensional cases

Here, we would like to extend the above results to higher orders of Adinkras. Particulary, we can do something similar for the more general case associated with  $(n > 2)$ . The corresponding compactification has been extensively studied to generate lower dimensional type IIA superstring. In this way, the Adinkras will be based on the polyvalent geometry in which the nodes are connected with more than two other ones as shown in the previous section associated with the trivalent geometry. The latter generalizes the bivalent one appearing in the case of the elliptic curve  $T^2$ . Based on this observation, we believe that the physics of the extremal black branes in type IIA superstring on  $T^n$  shears similarities with  $n$  qubits. Before giving a general picture, we illustrate the case of  $T^3$ . The corresponding compactification produces seven dimensional extremal black branes. Their extremal near-horizon geometries are given by

$$AdS_{p+2} \times S^{5-p}. \quad (4.8)$$

These asymptotically flat, static and spherical solutions are classified by the following fundamental solutions

1. seven dimensional black holes dual to black 3-branes,
2. seven dimensional black strings dual to black 2-branes.

The seven dimensional correspondence, in the presence of the D3-brane, can be illustrated in table 2.

The same analysis can be done for the extremal black hole in type IIA superstring on  $T^n$ . In this case, the near horizon geometry of the extremal black holes reduces to

$$AdS_2 \times S^{8-n}. \quad (4.9)$$

Using the fact that the real Hodge diagram of  $T^n$  can encode the information on the black brane charges and  $n$  qubit systems, we can give a possible brane configuration producing extremal black branes in  $10 - n$  dimensions, obtained from the compactification of type IIA superstring

$T^3$	Adinkra	qubit system	Black hole system
1	(000)	$ 000\rangle$	D0-brane
$dx_1$	(001)	$ 001\rangle$	F-string
$dx_2$	(010)	$ 010\rangle$	F-string
$dx_3$	(100)	$ 100\rangle$	F-string
$dx_1dx_2$	(011)	$ 011\rangle$	D2-brane
$dx_2dx_3$	(110)	$ 110\rangle$	D2-brane
$dx_1dx_3$	(101)	$ 101\rangle$	D2-brane
$dx_1dx_2dx_3$	(111)	$ 111\rangle$	D3-brane (T-dual of D2-brane)

Table 2: This table gives the correspondence between eight dimensional black hole, Adinkra and qubit systems.

on  $T^n$ . Similar discussion be also done for the extremal black branes. For illustration, the general picture for the black holes can be summarized in table 3.

$T^n$	Adinkra	qubit system	Black hole system
$\prod_{\ell=1}^n (\overline{e}_\ell + e_\ell dx_\ell)$	$(e_1, \dots, e_n)$	$ e_1, \dots, e_n\rangle$	$k$ -brane

Table 3: This table gives the correspondence between 10- $n$  dimensional black holes, regular Adinkras and  $n$  qubit systems.

## 5 Odd and even geometries on $T^{n|n}$ and superqubits

Having discussed the bosonic qubits, it is very natural to consider superqubit systems and supersymmetric Adinkras in the above established correspondence. In fact, superqubits have been investigated in connection with Lie supersymmetries[22, 23, 29]. Roughly speaking, a superqubit can take three values: 0 or 1 or  $\bullet$ . In fact, 0 and 1 are bosonic while  $\bullet$  is fermionic. In this section, we investigate the superqubits in the framework of the toroidal compactification with bosonic and fermionic coordinates. In particular, we explore the superform cohomology of a particular real supermanifold  $T^{n|m}$  equipped with  $n$  bosonic coordinates  $x_i$  and  $m$  fermionic coordinates  $\theta_\alpha$ . It is recalled that the supermanifolds have been extensively studied in string theory and related topics including mirror geometries [45, 46, 47].

In what follows, we consider a special geometry where  $n = m$  corresponding to the real supermanifold  $T^{n|n}$  and make contact with superqubits. More precisely, we would like to elaborate the real Hodge diagram of  $T^{n|n}$ . Up some assumptions specified later on, we show that a part of

this extended real Hodge diagram can be explored to represent  $n$  superqubits using the supersymmetric Adinkras involving bosonic and fermionic nodes. In fact, these supersymmetric real Hodge diagrams can be built using the extension of the above binary notation. For simplicity reason and keeping the contact with the previous section, we use the notation  $h^{e_1, \dots, e_n | \alpha_1, \dots, \alpha_n}$  associated with the following differential superform

$$h^{e_1, \dots, e_n | \alpha_1, \dots, \alpha_n} \longrightarrow \prod_{\ell=1}^n (\bar{e}_\ell + e_\ell dx_\ell) \prod_{\alpha=1}^n (\bar{e}_\alpha + e_\alpha d\theta_\alpha) \quad (5.1)$$

Its degree is given by

$$d = \sum_{\ell=1}^n e_\ell + \sum_{\alpha=1}^n e_\alpha. \quad (5.2)$$

To give a differential geometry representation of  $n$  superqubits, we postulate the following constraints on the superform degrees:

- the lowest degree is zero associated with a bosonic state
- the highest degree is  $n$  associated either with a bosonic or a fermionic state according to the parity of  $n$ .

More precisely, we will be interested in the following superforms on the real supermanifold  $T^{n|n}$

$$\prod_{\ell=1}^n (\bar{e}_\ell + e_\ell dx_\ell) \prod_{\alpha=1}^{n-\ell} (\bar{e}_\alpha + e_\alpha d\theta_\alpha). \quad (5.3)$$

Based on the above assumption, combinatorial calculation reveals that the total number of the associated cycles is given by

$$\sum_{\ell=0}^n C_n^l \sum_{k=0}^{n-\ell} C_{n-\ell}^k. \quad (5.4)$$

It is worth noting that if we put  $\ell = 0$ , we recover the bosonic case as discussed in section 2. Using the expression of the Taylor series  $(1+x)^n$ , we can show that this number is exactly the state number of  $n$  superqubits

$$\sum_{\ell=0}^n C_n^l \sum_{k=0}^{n-\ell} C_{n-\ell}^k = 3^n. \quad (5.5)$$

This includes the bosonic and the fermionic states. To get the state number of each sector, we use the following splitting

$$3^n = \frac{3^n - 1}{2} + \frac{3^n + 1}{2}. \quad (5.6)$$

The calculation shows the following relations

$$\sum_{\ell=\text{even}}^n C_n^l \sum_{k=0}^{n-\ell} C_{n-\ell}^k = \frac{3^n + 1}{2} \quad (5.7)$$

$$\sum_{\ell=\text{odd}}^n C_n^l \sum_{k=0}^{n-\ell} C_{n-\ell}^k = \frac{3^n - 1}{2}. \quad (5.8)$$

The number of the bosonic and the fermionic states can be identified by exploring the following formal supersymmetry structure

$$\begin{aligned} BB &= B \\ BF &= F \\ FB &= F \\ FF &= B \end{aligned}$$

where  $B$  and  $F$  denote the bosonic and the fermionic generators respectively. An inspection shows that the number of the bosonic states is given

$$\text{Number of bosonic states} = \frac{3^n + 1}{2}. \quad (5.9)$$

Similarly, the number of fermionic states reads as

$$\text{Number of fermionic states} = \frac{3^n - 1}{2}. \quad (5.10)$$

In connection with graph theory, the corresponding Adinkras should be formed by  $\frac{3^n+1}{2}$  bosonic nodes and  $\frac{3^n-1}{2}$  fermionic nodes associated with even and odd geometries on  $T^{n|n}$  respectively. In this way, the link can be put as follows

$$\text{Bosonic nodes} \longrightarrow \text{Bosonic forms} \quad (5.11)$$

$$\text{Fermionic nodes} \longrightarrow \text{Fermionic forms}. \quad (5.12)$$

To illustrate this analysis, we examine the real supermanifold  $T^{2|2}$  associated with two superqubits. In particular, it is obvious to see that the bosonic states correspond to the following differential form

$$1, \quad dx_1, \quad dx_2, \quad dx_1 dx_2, \quad d\theta_1 d\theta_2. \quad (5.13)$$

The number of the corresponding bosonic states is  $\frac{3^2+1}{2} = 5$ . Similarly, we can get the fermionic states associated with

$$d\theta_1, \quad d\theta_2, \quad dx_1 d\theta_2, \quad d\theta_1 dx_2. \quad (5.14)$$

The number is  $\frac{3^2-1}{2} = 4$ . Motivated by the existence the supermanifolds and brane geometries, the corresponding Adinkras and brane charges will be represented in table 4. The table will include superbranes living in supermanifolds.

We can construct many additional examples by considering D3-branes and their supersymmetric versions.

$T^{2 2}$	Adinkra	qubit system	black hole system
1	(00 00)	$ 00\rangle 00\rangle$	Bosonic D0-brane
$dx_1$	(10 00)	$ 10\rangle 00\rangle$	Bosonic F-string
$dx_2$	(01 00)	$ 01\rangle 00\rangle$	Bosonic F-string
$dx_1 dx_2$	(11 00)	$ 11\rangle 00\rangle$	Bosonic D2-brane
$d\theta_1$	(00 10)	$ 00\rangle \bullet 0\rangle$	Fermionic F-string
$d\theta_2$	(01 00)	$ 01\rangle 0\bullet\rangle$	Fermionic F-string
$d\theta_1 dx_2$	(01 10)	$ 01\rangle \bullet 0\rangle$	Fermionic F-string
$d\theta_2 dx_1$	(10 01)	$ 10\rangle 0\bullet\rangle$	Fermionic D2-brane
$d\theta_1 d\theta_2$	(00 11)	$ 00\rangle \bullet\bullet\rangle$	Bosonic D2-brane

Table 4: This table gives an illustrated model based on the real supermanifold  $T^{2|2}$ .

## 6 Conclusion and discussions

In this paper, we have reconsidered the study of the extremal black branes in the framework of quantum information using graph theory based on the so called Adinkras. In particular, we have elaborated a one to one correspondence between qubit systems, Adinkras and extremal black branes embedded in maximally supergravity obtained from a low energy limit of type IIA superstring compactified on  $T^n$ . It has been observed that the physical states of  $n$  qubit systems can be represented graphically using Adinkras. These graphs are based on polyvalent geometries appearing in the case of Dynkin diagrams of Lie algebras. In this representation, the  $n$ -qubits are associated with the  $n$ -valent geometry in which each node is connected with  $n$  colored lines. Based on this observation, the  $n$  qubit is represented by a graph of  $2^n$  bosonic nodes connected by the colored  $n$  polyvalent geometry. In this graphic representation, each node encodes information on the qubit quantum states and the charges of the extremal black branes embedded in type IIA superstring on  $T^n$ . Then, we have proposed a possible generalization to  $n$  superqubits. More precisely, we have shown that these systems can be associated with odd and even geometries on the real supermanifold  $T^{n|n}$ . More precisely, the number of the corresponding bosonic and fermionic states are obtained using a combinatorial calculation.

Our paper comes up with many open questions related to quantum information theory. In fact, many concepts have been developed in such a theory including gates, circuits and entanglement states. It should be of interest to investigate such concepts using the graph theory and the physics dealing with black objects. Moreover, there have been many works trying to connect black hole with quantum information [48]. It should be interesting to make contact with such activities. This will be addressed elsewhere.

On Other hand, the analysis presented here might be extended in the case of the Calabi-

Yau manifolds. To speculate on this extension, let us consider the compactification of type IIA superstring on the K3 surface [49, 50]. It is recalled the the corresponding complex Hodge diagram reads as

$$\begin{array}{ccccccc}
& & h^{0,0} & & & & 1 \\
& & & & h^{0,1} & & 0 & 0 \\
h^{2,0} & & h^{1,1} & & h^{0,2} & = & 1 & 20 & 1 \\
& & h^{2,1} & & h^{1,2} & & 0 & 0 \\
& & & & h^{2,2} & & & & 1
\end{array}$$

It has been shown that the moduli space of such a compactification is given by the following factorization

$$\frac{SO(4, 20)}{SO(4) \times SO(20)} \times SO(1, 1). \quad (6.1)$$

It is observed that this factorization is linked to two possible black object solutions in six dimensions which are given by

$$\begin{aligned}
p = 0 : & \quad \text{corresponding to the black holes with a near-horizon geometry: } AdS_2 \times S^4 \\
p = 1 : & \quad \text{associated with a black string having a near-horizon geometry: } AdS_3 \times S^3.
\end{aligned}$$

In fact, the factor  $\frac{SO(4,20)}{SO(4) \times SO(20)}$  is associated with 24 black hole charges identified with entries appearing in the above complex Hodge diagram. The corresponding brane realization can be given by

$$\begin{array}{ccccccc}
1 & & & & D0 & & \\
& & & & & & \\
1 & 20 & 1 & = & D2 & D2 & D2 \\
& & & & & & \\
1 & & & & D4 & & 
\end{array} \quad (6.2)$$

At first sight, the brane representation could be related to the qudit systems [51]. However, we believe that this connection deserves a deeper study.

**Acknowledgements:** It is a pleasure to thank Prof N. Askour for discussions. AS is supported by the Spanish MINECO (grants FPA2009-09638 and FPA2012-35453) and DGIID-DGA (grant 2011-E24/2).

## References

- [1] A. Strominger, C. Vafa, *Microscopic Origin of the Bekenstein-Hawking Entropy*, Phys.Lett. **B379** (1996) 99-104, [arXiv:hep-th/9601029](#).

- [2] C. Vafa, *Black Holes and Calabi-Yau Threefolds*, Adv.Theor.Math.Phys.**2** (1998) 207-218, [hep-th/9711067](#).
- [3] J. Maldacena, A. Strominger, E. Witten, *Black Hole Entropy in M-Theory*, JHEP 9712 (1997)002, [arXiv:hep-th/9711053](#).
- [4] R. Ahl Laamara, A. Belhaj, L. B. Drissi, E. H. Saidi, *Black Holes in type IIA String on Calabi-Yau Threefolds with Affine ADE Geometries and q-Deformed 2D Quiver Gauge Theories*, Nucl. Phys. **B776** (2007) 287-326, [hep-th/0611289](#).
- [5] S. Ferrara, R. Kallosh and A. Strominger, N = 2 extremal black holes, Phys. Rev. D52 (1995) 5412, [hep-th/9508072](#). A. Strominger, Macroscopic entropy of N = 2 extremal black holes, Phys. Lett. B383, 39 (1996), [hep-th/9602111](#). S. Ferrara and R. Kallosh, Supersymmetry and attractors, Phys. Rev. D54, 1514 (1996), [hep-th/9602136](#). S. Ferrara and R. Kallosh, Universality of supersymmetric attractors, Phys. Rev. D54, 1525 (1996), [hep-th/9603090](#). S. Ferrara, G. W. Gibbons and R. Kallosh, Black Holes and Critical Points in Moduli Space, Nucl. Phys. B500 (1997) 75, [hep-th/9702103](#).
- [6] H. Ooguri, A. Strominger, C. Vafa, *Black Hole Attractors and the Topological String*, Phys. Rev. **D70** (2004) 106007, [hep-th/0405146](#).
- [7] S. Ferrara and R. Kallosh, *Supersymmetry and Attractors*, Phys. Rev. **D54** (1996) 1514, [hep-th/9602136](#).
- [8] A. Ceresole, S. Ferrara and A. Marrani, 4d/5d Correspondence for the Black Hole Potential and its Critical Points, Class. Quant. Grav. 24, 5651 (2007), [arXiv:0707.0964 \[hep-th\]](#).
- [9] S. Bellucci, S. Ferrara, A. Marrani and A. Yeranyan, Mirror Fermat Calabi-Yau threefolds and Landau-Ginzburg Black Hole Attractors, Riv. Nuovo Cim. 029, 1 (2006), [hep-th/0608091](#)
- [10] K. Saraikin and C. Vafa, Non-supersymmetric black holes and topological strings, Class. Quant. Grav. 25 (2008) 095007, [hep-th/0703214](#).
- [11] S. Bellucci, S. Ferrara, R. Kallosh, A. Marrani, *Extremal Black Hole and Flux Vacua Attractors*, Lect. Notes Phys. **755**(2008)115-191, [arXiv:0711.4547 \[hep-th\]](#).
- [12] A. Sen, *Black Hole Entropy Function, Attractors and Precision Counting of Microstates*, Gen. Rel. Grav. **40**(2008)2249-2431, [arXiv:0708.1270 \[hep-th\]](#).
- [13] A. Dabholkar, *Black hole entropy and attractors*, Class. Quant. Grav. **23** (2006) 957-980.



- [14] P. K. Tripathy, S. P. Trivedi, *Non-supersymmetric attractors in string theory*, JHEP **0603** (2006) 022, [hep-th/0511117](#).
- [15] A. Belhaj, L. B. Drissi, E. H. Saidi, A. Segui,  *$N=2$  Supersymmetric Black Attractors in Six and Seven Dimensions*, Nucl. Phys. **B796** (2008) 521-580, [arXiv:0709.0398](#).
- [16] R. Ahl Laamara, M. Asorey, A. Belhaj, A. Segui, *Extremal Black Brane Attractors on The Elliptic Curve*, J.Phys. **A43**(2010) 105401, [arXiv:0907.0093](#).
- [17] A. Belhaj, *On Black Objects in Type IIA Superstring Theory on Calabi-Yau Manifolds*, African Journal Of Math. Phys. Vol. **6**(2008)49-54, [arXiv:0809.1114](#) [[hep-th](#)].
- [18] S. Ferrara, A. Marrani, J. F. Morales, H. Samtleben, *Intersecting Attractors*, [arXiv:0812.0050](#) [[hep-th](#)].
- [19] M. J. Duff, String triality, black hole entropy and Cayleys hyperdeterminant, Phys. Rev. D **76** (2007) 025017, [hep-th/0601134](#).
- [20] M. J. Duff and S. Ferrara, Black hole entropy and quantum information, Lect. Notes Phys. **755** (2008) 93, [hep-th/0612036](#).
- [21] L. Borsten, M. J. Duff, P. Lévy, *The black-hole/qubit correspondence: an up-to-date review*, [arXiv:1206.3166](#).
- [22] L. Borsten, M.J. Duff, A. Marrani, W. Rubens, *On the Black-Hole/Qubit Correspondence*, Eur.Phys.J.Plus **126** (2011) 37, [arXiv:1101.3559](#).
- [23] M. J. Duff, S. Ferrara, *Four curious supergravities*, Phys.Rev. **D83** (2011) 046007, [arXiv:1010.3173](#).
- [24] P. Lévy, *Qubits from extra dimensions*, Phys. Rev. **D 84** (2001) 125020.
- [25] L. Borsten, D. Dahanayake, M.J. Duff, W. Rubens, H. Ebrahim, Freudenthal triple classification of three-qubit entanglement, Phys. Rev. A **80** (2009) 032326, [arXiv:0812.3322](#) [[quant-ph](#)].
- [26] L. Borsten, D. Dahanayake, M. J. Duff, A. Marrani, W. Rubens, Four-qubit entanglement classification from string theory, Phys. Rev. Lett. **105**, 100507 (2010), [arXiv:1005.4915](#) [[hep-th](#)].
- [27] L. Borsten, D. Dahanayake, M.J. Duff, W. Rubens, Superqubits, Phys. Rev. D **81** (2010) 105023, [arXiv:0908.0706](#) [[quant-ph](#)].

- [28] L. Castellani, P. A. Grassi, L. Sommovigo, Triality Invariance in the N=2 Superstring, Phys. Lett. B678 (2009) 308, arXiv:0904.2512 [hep-th].
- [29] L. Borsten, K. Bradler, M. J. Duff, *Tsirelson's bound and supersymmetric entangled states*, arXiv:1206.6934.
- [30] L. Castellani, P. A. Grassi, L. Sommovigo, *Quantum Computing with Superqubits*, arXiv:1001.3753.
- [31] P. Lévy, G. Sárosi, *Hitchin functionals are related to measures of entanglement*, Phys. Rev. **D86** (2012) 105038.
- [32] S. James Gates, Jr., Kory Stiffler, *Adinkra 'Color' Confinement In Exemplary Off-Shell Constructions Of  $4D$ ,  $\mathcal{N} = 2$  Supersymmetry Representations*, arXiv:1405.0048.
- [33] Y. X Zhang, Adinkras for Mathematicians, arXiv:1111.6055
- [34] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, New York, NY, USA, 2000.
- [35] D. R. Terno, *Introduction to relativistic quantum information*, arXiv:quant-ph/0508049.
- [36] M. Kargarian, *Entanglement properties of topological color codes*, Phys. Rev. A 78 (2008)062312, arXiv:0809.4276.
- [37] B. L. Douglas, S. James Gates Jr., Jingbo B. Wang, *Automorphism Properties of Adinkras*, arXiv:1009.1449
- [38] C.F. Doran, M.G. Faux, S.J. Gates, Jr., T. Hubsch, K.M. Iga, G.D. Landweber, *Adinkras and the Dynamics of Superspace Prepotentials*, arXiv:hep-th/0605269.
- [39] C.F. Doran, M.G. Faux, S.J. Gates, Jr., T. Hubsch, K.M. Iga, G.D. Landweber, *On Graph-Theoretic Identifications of Adinkras, Supersymmetry Representations and Superfields*, Int.J.Mod.Phys.**A22**(2007)869-930,2007, arXiv:math-ph/0512016.
- [40] M. Faux, S. J. Gates Jr, *Adinkras: A Graphical Technology for Supersymmetric Representation Theory* Phys.Rev. **D71** (2005) 065002, arXiv:hep-th/0408004.
- [41] C. Doran, K. Iga, G. Landweber, *An application of Cubical Cohomology to Adinkras and Supersymmetry Representations*, arXiv:1207.6806.
- [42] A. Salam and E. Sezgin, *Supergravities in diverse Dimensions*, Edited by A. Salam and E. Sezgin, North-Holland, World Scientific **1989**, vol.1.
- [43] K. Intriligator and N. Seiberg, Phys. Lett. **B387** (1996) 513.

- [44] K. Hori, H. Ooguri and C. Vafa, Nucl. Phys. **B504** (1997) 147.
- [45] E. Witten, *Notes On Supermanifolds and Integration*, [arXiv:1209.2199](#).
- [46] M. Aganagic, C. Vafa, *Mirror symmetry and supermanifolds*, [hep-th/0403192](#).
- [47] A. Belhaj, L.B. Drissi, J. Rasmussen, E.H. Saidi, A. Sebbar, *Toric Calabi-Yau supermanifolds and mirror symmetry*, J.Phys. **A38** (2005) 6405-6418, [arXiv:hep-th/0410291](#).
- [48] T. Prudencio, D. J. Cirilo-Lombardo, E. O. Silva, H. Belich, Black hole qubit correspondence from quantum circuits, e-Print: [arXiv:1401.4196](#) [quant-ph].
- [49] B.R. Greene, String Theory on Calabi Yau Manifolds, [hep-th/9702155](#).
- [50] P. Aspinwall, K3 surfaces and String Duality, [hep-th/961117](#)
- [51] M. Rios, *Extremal Black Holes as Qudits*, [arXiv:1102.1193](#).